Discussion 10 Worksheet

Global maxima and minima

Date: 9/24/2021

MATH 53 Multivariable Calculus

1 Finding Extrema

Find the global maxima and minima of the following functions on their indicated domains.

1. The function $f(x, y) = x^2 - y$ on the domain $D = [0, 2] \times [0, 2]$.
2. The function $f(x, y) = x - y$ on the domain $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$.
3. The function $f(x, y) = x^2 - xy + y^2 - 3y$ on the region bounded by the x and y axes and the line $x + y = 4$.

1.1 Past midterm problems

1. Find the following. If an expression is undefined, say so.
   - dy/dx, where $x = 2\sin t$, $y = 3\cos t$. Express your answer as a function of t.
   - The area of the region between the curve whose expression in polar coordinates is $r = e^\theta$ $(0 \leq \theta \leq \pi/2)$, the line $\theta = 0$, and the line $\theta = \pi/2$.
   - $\lim_{(x,y)\to(0,0)} x/y$.
   - The equation of the plane tangent to the surface $z = (x + y)^{1/2}$ at the point where $x = 2, y = 7$.
   - $$\frac{\partial^2}{\partial x \partial y}(f(x)g(y)),$$
     where f and g are differentiable functions. (Express your answer in terms of f and g and their derivatives.)
   - $$\int_0^1 \vec{\jmath} \times (t^2\vec{\imath} + e^{-t^2}\vec{\jmath} + (\tan t)\vec{k})dt$$
     where $\vec{\imath}$, $\vec{\jmath}$, and $\vec{k}$ are the standard basis vectors in $\mathbb{R}^3$.
2. - Use the cross product to find numbers p, q, r, and s such that the plane $px+qy+rz+s = 0$ goes through the points $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 3)$.
   - Now find a DIFFERENT set of numbers p', q', r', and s' such that the plane $p'x + q'y + r'z + s' = 0$ still goes through these three points.
3. Consider the curve described in polar coordinates by $r = 2 + \cos 2\theta$.
   - Explain, without doing any computation, why the area enclosed by the curve must be less than 9π.

- Compute the area enclosed by the curve.
- Sketch the curve.

Note: These problems are taken from the worksheets for Math 53 in the Spring of 2021 with Prof. Stankova.